Mathematics: analysis and approaches
Higher level
Paper 2

15 May 2026

Zone A morning | **Zone B** morning | **Zone C** morning

Session number

2 hours

Instructions to students

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following table shows the number of books read by a group of students at a certain school in a month.

Books read	Frequency
0	3
1	8
2	12
3	15
4	10
5	7
6	4
7	1

(a) Write down the mode. [1]

(b) Find

(i) the mean;

(ii) the median;

(iii) the standard deviation. [3]

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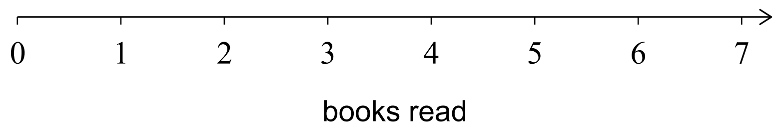
(Question 1 continued)

(c) Draw a box and whisker diagram that represents the number of books read on the scale below. [2]



2. [Maximum mark: 6]

A closed cylindrical container has base radius $r\,\text{cm}$ and height $h\,\text{cm}$. The container has a volume of $500\,\text{cm}^3$.

(a) Show that $h = \dfrac{500}{\pi r^2}$. [1]

Let the total external surface area of the cylinder be $A\,\text{cm}^2$.

(b) Show that $A = 2\pi r^2 + \dfrac{1000}{r}$. [2]

(c) By using a graph, or otherwise,

(i) find the minimum value of A;

(ii) write down the corresponding value of r. [3]

3. [Maximum mark: 5]

The velocity, $v\,\mathrm{m\,s^{-1}}$, of a particle at time t seconds, is given by $v = 10\left(\frac{t^2}{5} - \mathrm{e}^{-0.2t}\right)$ for $t \geq 0$.

(a) Find the initial velocity of the particle. [1]

(b) Find the distance travelled by the particle in the first five seconds. [2]

(c) Find the time when the acceleration of the particle is $19.3\,\mathrm{m\,s^{-2}}$. [2]

4. [Maximum mark: 6]

Consider a binomial random variable, X, such that $X \sim \mathrm{B}(15, p)$, where $p \neq 0$.

The mean of the distribution is three times as large as the standard deviation.

(a) Find p. [3]

(b) Find $\mathrm{P}(X = 9 \mid X > 6)$. [3]

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5. [Maximum mark: 5]

Part of the curve of $y = \dfrac{20}{x+1}$ between point $\mathrm{A}(1\,,\,10)$ and point $\mathrm{B}(4\,,\,4)$ is rotated $360°$ about the y-axis to form a solid of revolution.

Find the volume of the solid formed.

6. [Maximum mark: 5]

A continuous random variable X has a probability density function given by

$$f(x)=\begin{cases}\dfrac{1-\cos 2x}{a}, & 0\le x\le \pi\\ 0 & \text{, otherwise}\end{cases}\quad \text{where } a>0.$$

(a) Find the exact value of a. [2]

(b) Find $\mathrm{Var}(X)$. [3]

7. [Maximum mark: 6]

Consider the following system of linear equations

$$2x - y + z = 1$$
$$2x - 3y - z = 3$$
$$4x - 3y + z = k$$

(a) Find the value of k for which the system of equations has an infinite number of solutions. [3]

For the value of k found in part (a),

(b) (i) interpret this result geometrically;

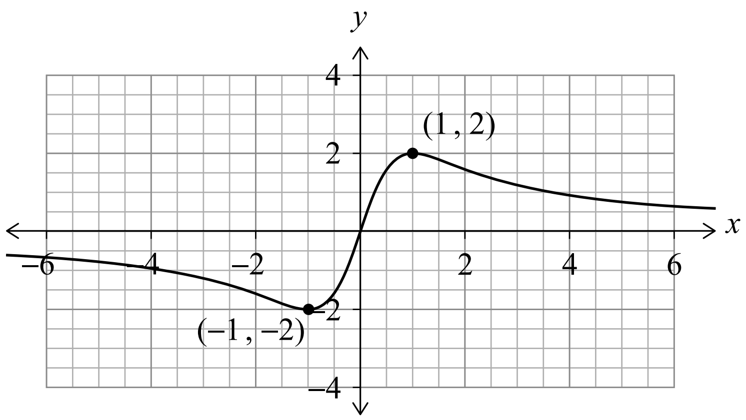
(ii) express the solution to the system of equations in parametric form. [3]

8. [Maximum mark: 8]

Consider the function defined by $f(x) = \dfrac{ax}{bx^2 + c}$ for $x \in \mathbb{R}$, where $a, b, c > 0$.

(a) Show algebraically that $f(x)$ is an odd function. [2]

The graph of $y = f(x)$ is shown below. Point $(1, 2)$ is a local maximum and point $(-1, -2)$ is a local minimum.



(b) (i) Show that $b = c$ and $a = 4b$.

(ii) Hence, deduce that $f(x) = \dfrac{4x}{x^2 + 1}$. [6]

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9. [Maximum mark: 8]

(a) Show that $\frac{\cos x}{(1-\sin x)(1+\sin x)} \equiv \sec x$, where $x \neq (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$. [2]

(b) Use the result from part (a) and the substitution $u = \sin x$ to find $\int \sec x \, dx$. [6]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 15]

The weights of red potatoes are normally distributed with mean 173 grams and standard deviation 21 grams.

(a) Find the probability that a randomly selected red potato weighs

(i) less than 180 grams;

(ii) between 180 grams and 200 grams;

(iii) between 180 grams and 200 grams given it weighs more than 180 grams. [6]

The weights of purple potatoes are normally distributed with mean μ grams and standard deviation σ grams.

The probability that a purple potato weighs less than 170 grams is 0.62102.

The probability that a purple potato weighs between 170 grams and 180 grams is 0.23799.

(b) Find the mean, μ , and standard deviation, σ , of purple potatoes. [6]

Joli has one bag of red potatoes and one bag of purple potatoes.

Joli randomly selects one potato from each bag.

(c) Find the probability that at least one of these potatoes weighs more than 180 grams. [3]

Do **not** write solutions on this page.

11. [Maximum mark: 21]

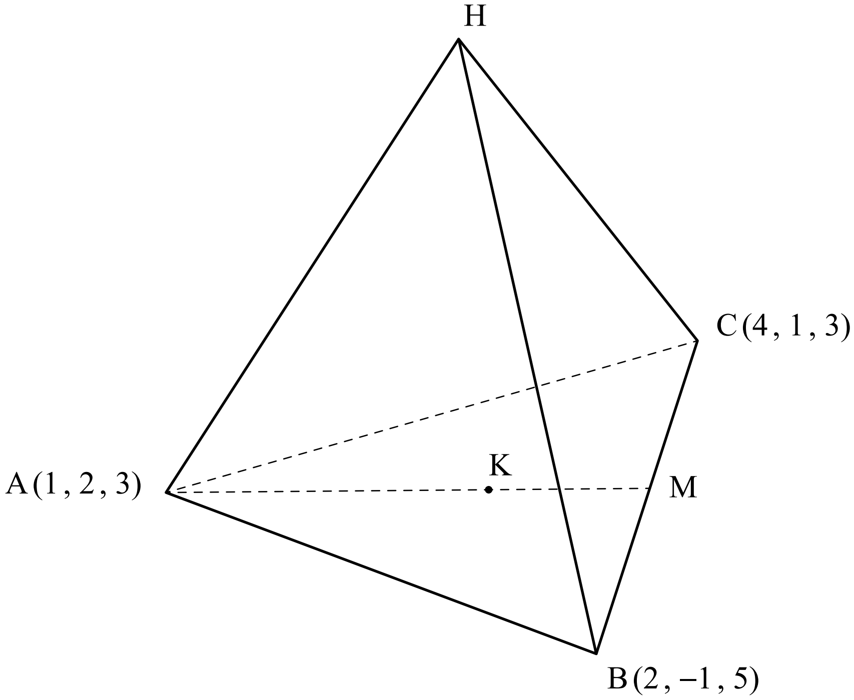
Consider the points $\mathrm{A}(1, 2, 3)$, $\mathrm{B}(2, -1, 5)$ and $\mathrm{C}(4, 1, 3)$.

(a) (i) Find $\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}$.

(ii) Hence, or otherwise, find the area of triangle ABC. [5]

The points A, B and C form the triangular base of a right-pyramid with vertex H. This is shown in the following diagram.

diagram not to scale



M is the midpoint of $[\mathrm{BC}]$ and point K lies on $[\mathrm{AM}]$ such that $\overrightarrow{\mathrm{AK}} = \frac{2}{3}\overrightarrow{\mathrm{AM}}$.

(b) Show that K has coordinates $\left(\frac{7}{3}, \frac{2}{3}, \frac{11}{3}\right)$. [3]

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(Question 11 continued)

The plane Π contains the points A, B and C.

The Cartesian equation of Π is $x + 3y + 4z = 19$.

The line L_1 is perpendicular to Π and passes through K.

(c) Write down a vector equation of L_1 in the form $\boldsymbol{r} = \boldsymbol{p} + s\boldsymbol{d}$, where $s \in \mathbb{R}$. [1]

The point $\mathrm{H}\left(2, -\frac{1}{3}, \frac{7}{3}\right)$ lies on L_1.

(d) Find the volume of the pyramid ABCH. [3]

The line L_2 has vector equation $r = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix}$ where $t \in \mathbb{R}$.

Let α be the acute angle that L_2 makes with Π.

(e) Find α, correct to the nearest degree. [3]

(f) Show that the lines L_1 and L_2 are skew. [6]

Do **not** write solutions on this page.

12. [Maximum mark: 19]

Consider the function $f(x) = 2\arctan(x-1) + \frac{\pi}{2}$ where $x \in \mathbb{R}$.

(a) Sketch the graph of $y = f(x)$, clearly indicating any asymptotes with their equations. [3]

(b) (i) Describe a sequence of transformations that transform the graph of $y = \arctan x$ to the graph of $y = 2\arctan(x-1) + \frac{\pi}{2}$ for $x \in \mathbb{R}$.

(ii) Write down the coordinates of the point of inflexion on the graph of $y = f(x)$. [4]

(c) (i) Find $f^{-1}(x)$ and state its domain.

(ii) Find the coordinates of the point of intersection of the graphs $y = f(x)$ and $y = f^{-1}(x)$ for $x > 0$. [6]

(d) Find the equation of the tangent to the graph of $y = f(x)$ at $x = 3$. [2]

This tangent intersects the graph of $y = f(x)$ at the point P.

(e) (i) Find the coordinates of P.

(ii) Find the area enclosed by the tangent and the graph of $y = f(x)$. [4]